In the limit as $\epsilon \to 0$ with n_0 fixed, $N_0 = n_0/\epsilon \to \infty$, so that Eq. (28) becomes

$$p(s,N) = p_m(s,N) + \int_N^\infty \frac{h_N}{h^3}(\rho_m U_e^2 - \rho h^2 u^2)dN \quad (29)$$

Use of Eqs. (18) and (22) allow computation of $\partial p_m/\partial s$ as

$$\partial p_m/\partial s = -\rho_m(U_e/h)(\partial/\partial s)(U_e/h) \quad (30)$$

Equation Summary

Within the framework outlined previously, the governing system valid to second order can now be summarized as

continuity

$$(\partial/\partial s)(\rho r^j u) + (\partial/\partial N)(\rho h r^j v) = 0 \quad (31)$$

s-momentum

$$\rho[u\,(\partial u/\partial s) + v(\partial hu/\partial N)] - \rho_m(U_e/h)(\partial/\partial s)(U_e/h) + \frac{\partial}{\partial s}\left[\int_N^\infty \frac{h_N}{h^3}(\rho_m U_e^2 - \rho h^2 u^2)dN\right] = \frac{1}{hr^j}\frac{\partial}{\partial N}(r^j h^2 \tau) \quad (32)$$

where

$$\tau = \mu[\partial u/\partial N - (h_N/h)u] \quad (33)$$

energy

$$\rho\left(u\frac{\partial H}{\partial s} + hv\frac{\partial H}{\partial N}\right) = \frac{1}{r^j}\frac{\partial}{\partial N}[hr^j(q + u\tau)] \quad (34)$$

where

$$q = (\mu/\sigma)\partial T/\partial N = (\mu/\sigma)[\partial H/\partial N - u(\partial u/\partial N)] \quad (35)$$

and

$$\mu = f(T) \quad (36)$$

$f(T)$ represents any convenient temperature viscosity law with

$$T = H - u^2/2 \quad (37)$$

$$p_m + \int_N^\infty \frac{h_N}{h^3}(\rho_m U_e^2 - \rho h^2 u^2)dN = \frac{\gamma - 1}{\gamma}\rho\left(H - \frac{u^2}{2}\right) \quad (38)$$

and

$$p_m/p_e = (\rho_m/\rho_e)^\gamma = [1 + (U_e^2/2)/(H_e - U_e^2/2) \times (1 - 1/h^2)]^{\gamma/\gamma-1} \quad (39)$$

The edge conditions U_e and H_e are assumed known.

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Supersonic Combustion Tests with a Double-Oblique-Shock SCRAMjet in a Shock Tunnel

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Nomenclature

C_h = Stanton number, $C_h \approx \dot{q}_w/\rho_\infty U_\infty(H_o - H_w)$
C^* = form of Chapman-Rubesin viscosity-coefficient, $C^* = \mu^* T_\infty/\mu_\infty T^*$
E.R. = equivalence ratio (fuel-air ratio/stoichiometric fuel-air ratio)
H = total enthalpy
M = Mach number
P,p = pressure
$\dot{q}$ = heat-transfer rate
Re = Reynolds number
T = temperature
T^* = reference temperature, $T^* = (T_o/6)(1 + 3T_w/T_o)$
U = velocity
$\bar{v}_\infty{}^*$ = hypersonic interaction parameter, $\bar{v}_\infty{}^* = M_\infty(C^*/Re_\infty)^{1/2}$
X = distance

Subscripts

C = conditions in the combustor
ex = conditions at the exit plane of the combustor
o = reservoir (total)
SLR = sodium line reversal measurement
w = conditions evaluated at the wall temperature (300°K)
∞ = freestream condition

Superscripts

$'$ = Pitot value

I. Introduction

SOME results of a continuing research program to develop a capability for testing integrated SCRAMjets in the AEDC Gas Dynamic Tunnel F[1] (Hotshot) are presented. During this program, an integrated double-oblique-shock SCRAMjet model was developed to provide a supersonic combustion test bed. It was appreciated that a double-oblique-shock model is a very inefficient device for total pressure recovery for a SCRAMjet; however, the object of the test program was to develop a "simple" model in which supersonic combustion could be demonstrated to prove the feasibility of testing integrated SCRAMjet engines in a pulse facility, and that adequate instrumentation could be developed to provide meaningful data analysis.

Table 1 Typical test conditions

Driver gas temperature, °K	M_∞	Re_∞/ft	p_o, psia	T_o, °K	$p_{o,\infty}'$, psia	T_∞, °K	p_∞, psia
300	11	3.38×10^6	10^4	1860	18.0	88	0.12
480	10.9	2.22×10^6	1.13×10^4	2270	19.2	110	0.127

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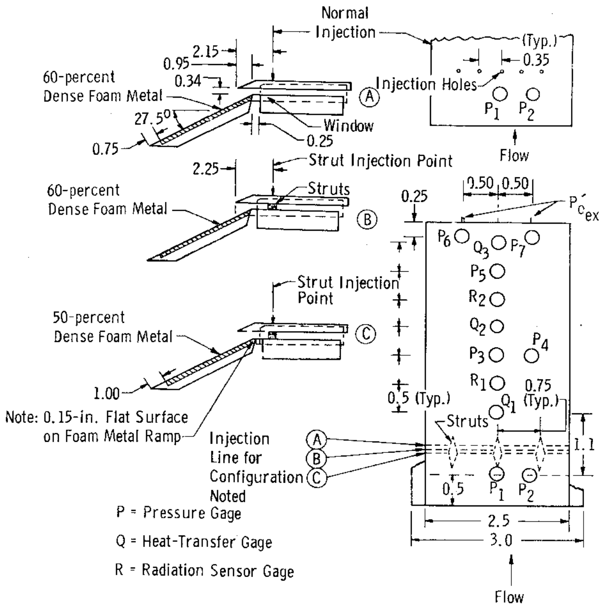

Fig. 1 Model configurations, instrumentation plate, and injection modes.

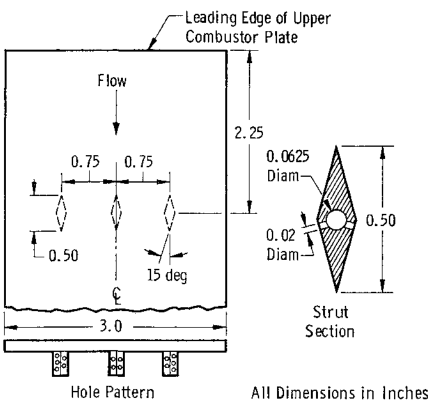

Fig. 2 Strut injector detail.

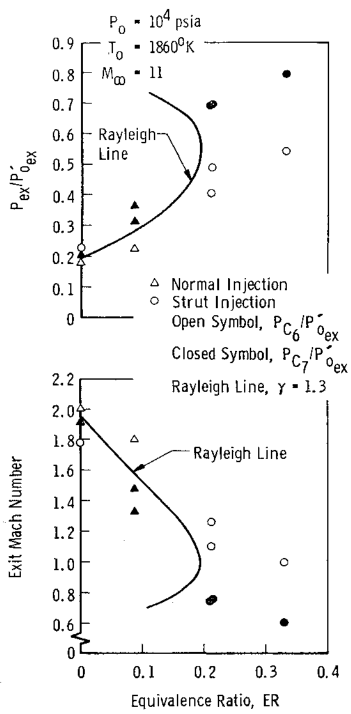

Fig. 3 Dimensionless combustor exit pressures vs equivalence ratio (cold driver).

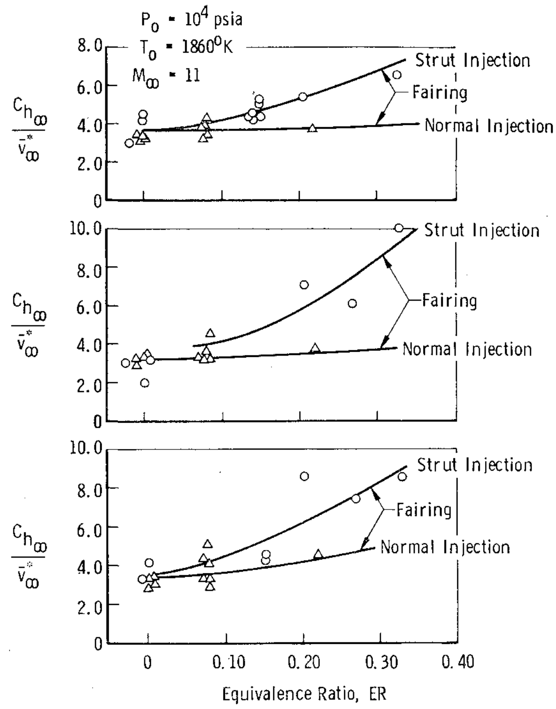

Fig. 4 Dimensionless combustor heat-transfer rates vs equivalence ratio (cold driver).

II. Test Apparatus

The development tests were conducted at the 16-in.-diam test section of Shock Tunnel I[2] at a nominal freestream Mach number of 11. Typical test conditions are given in Table 1. The useful test time was approximately 3 msec. Tests were conducted with shock tunnel driver gas (helium) temperatures of 300° and 480°K. The shock tunnel was operated in the tailored-interface mode. Model and fuel injector configurations are shown in Figs. 1 and 2. More detailed information on the model is given in Ref. 3.

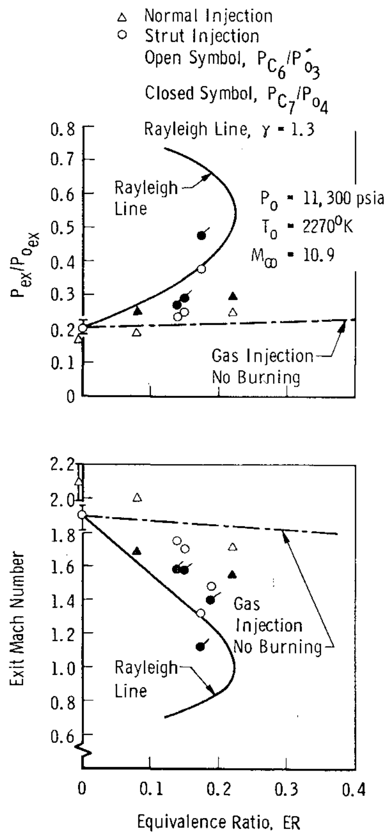

Fig. 5 Dimensionless combustor exit pressures vs equivalence ratio (hot driver).

Table 2 Inviscid two-shock-theory model conditions

Station	Free-stream	Ramp	Combustor
p, psia	0.12	4.8	27.3
T, °K	88	647	1160
M	11.0	3.5	2.1
U, fps	6800	5800	4600

III. Summary of Results

Tests without fuel injection

Considerable effort[3] was expended before an acceptable flow was obtained in the two-dimensional combustor. The primary source of difficulty was the interaction of the shock wave from the cowl lip with the inlet boundary layer and the Prandtl-Meyer expansion at the intersection of the inlet and combustor. This problem was compounded by significant source flow effects which caused a decay in static pressure along the inlet ramp surface and the finite ramp width (3 in.) which caused cross flows in the ramp boundary layer and was a possible cause of the early boundary-layer transition. Experimental heat-transfer rates to the ramp indicated that the end of transition from a laminar to a turbulent boundary layer occurred at approximately 3 in. from the plate leading edge, whereas the estimated length for the end of transition (using current transition literature) was approximately 6–7 in.

The measured average static pressure in the combustor was $\approx$13.5 psia and the average static temperature and Mach number inferred from measured values of static and Pitot pressures at the combustor exit were 1200°K and 1.9 respectively. Inviscid two-shock theory estimates are given in Table 2. Approximately 50% of the estimated pressure was obtained. Repeatable, steady operation of the model was obtained when a porous ramp was used to bleed off part of the inlet boundary layer and the cowl lip, positioned so that the second shock impinged as close as possible to the intersection of the ramp and combustor.

Tests with fuel injection

Hydrogen fuel, at 300°K total temperature, was injected from either sonic orifices in the upper combustor plate—normal injection—or sonic orifices in the struts (see Fig. 1). With a cold driver gas temperature (300°K), satisfactory combustion[4] was demonstrated with strut injection only, as shown in Figs. 3 and 4.

The driver gas was heated to 480°K which increased the total temperature and pressure as shown in Table 1. The combustor static pressure level increased from 13.5 to 18.2 psia and the static temperature increased from 1200° to 1530°K. The average Mach number at the combustor exit did not change significantly from the previous value of $\approx$1.9.

Hydrogen fuel at 300°K was again injected with satisfactory combustion[4] data obtained for strut injection tests only as shown in Figs. 5 and 6. The sodium-line-reversal measurements confirmed significant increases in static temperature due to combustion on strut injection tests only.

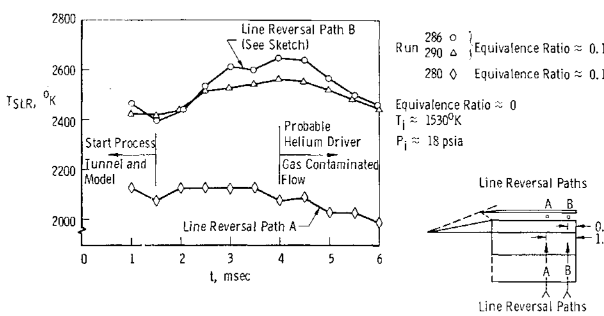

Fig. 6 Line reversal temperatures as a function of time.

IV. Conclusions

It has been demonstrated that it is possible to carry out supersonic combustion tests for development of instrumentation and analytical techniques applicable to SCRAMjets within a useful test time of approximately 3 msec in a shock tunnel.

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Flight Results Showing the Effect of Mass Addition on Base Pressure

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Nomenclature

P_b = base pressure
P_∞ = freestream pressure
P_b/P_∞ = base pressure ratio
Re_L = freestream Reynolds number based on axial length
r = radius at any point on the base
R = maximum base radius
M_∞ = freestream Mach number
$\dot{m}/\rho A V$ = mass addition parameter
$K_{\dot{m}}$ = ratio of base pressure with mass addition to base pressure with zero mass addition
$\Delta\theta$ = flow turning angle
ρ = freestream density
A = base area
V = freestream velocity
L_{Neck} = distance from R/V base to wake neck
R/V = re-entry vehicle

I. Introduction

Recent full scale re-entry flight test base pressure data in turbulent flow have shown a dependency on the heat shield material. The heat shield material, of course, determines the amount of mass addition to the boundary layer due to the ablation process. This present Note has a threefold purpose: 1) to present the flight test base pressure data results; 2) to propose a flow hypothesis mechanism which explains how mass addition affects base pressure; and, 3) to show correlations of the base pressure ratio data with mass addition rate.

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